Generation of entangled photon pairs from a single quantum dot embedded in a planar photonic-crystal cavity

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We present a formal theory of single quantum dot coupling to a planar photonic crystal that supports quasidegenerate cavity modes and use this theory to describe and optimize entangled-photon pair generation via the biexciton-exciton cascade. In the generated photon pairs, either both photons are spontaneously emitted from the dot or one photon is emitted from the biexciton spontaneously while the other is emitted via the leaky cavity mode. In the strong-coupling regime, the generated photon pairs can be maximally entangled in qualitative agreement with the dressed-state predictions of Johne *et al.* [Phys. Rev. Lett. **100**, 240404 (2008)]. We derive useful analytical formulas for the spectrum of the emitted photon pairs in the presence of exciton and biexciton broadening, which is necessary to connect to realistic experiments and demonstrate the important differences with the approximate dressed-state approach. We also present a method for calculating and optimizing the entanglement between the emitted photons, which can account for postsample spectral filtering. Pronounced entanglement values of greater than 80% are demonstrated using experimentally achievable parameters even without spectral filtering.

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I. INTRODUCTION

A source of polarization-entangled photon pairs has wide uses in quantum optics,[1–3] leading to applications such as quantum computation,[4,5] quantum information processing,[6,7] quantum cryptography,[8] and quantum metrology.[9] Most of the experiments demonstrated to date employ entangled photons generated by parametric down conversion (PDC).[10,11] A PDC is a "heralded" source of entangled photons in which the number of generated photon pairs is probabilistic. However, in many experiments, particularly in quantum information processing,[12] a deterministic scalable source of entangled photons is essential. Recently, there has been considerable interest in developing an all-solid-state "on demand" source of entangled photon pairs using single quantum dots (QDs).[13–18] In QDs, entangled photon pairs can be generated in a biexciton cascade decay via exciton states of angular momenta +1 and −1; single QDs are particularly appealing since they are fixed in place, scalable, and have long coherence times. However, a major difficulty for implementing these schemes is the naturally occurring anisotropic energy difference between the exciton states of different angular momentum.[19] Specifically, a small anisotropic energy difference can make the emitted x-polarized and y-polarized photon pairs distinguishable and thus the entanglement between the photons is largely wiped out. Several significant efforts have been made to overcome this problem, for example, by spectrally filtering indistinguishable photon pairs,[13] by applying external fields to make the exciton states degenerate,[14,15] by thermal annealing of QDs,[16] by selecting QDs with smaller anisotropic energy difference,[17] and by using temporal gates.[18] There have been a few other interesting proposals by suppressing the biexciton binding energy in combination with time reordering.[20,21]

Recently, Johne *et al.*[22] proposed an interesting cavity-QED scheme in the strong-coupling regime where the exciton states become dressed with the cavity field and form polariton states.[23] The lifetimes of the polariton states are much smaller than the lifetimes of excitons, which minimizes the effects of dephasing.[24,25] In the last few years a number of experimental groups have demonstrated the strong-coupling regime using single QDs integrated with planar photonic-crystal cavities.[26–28] These emerging "on-chip" cavity structures form an important breakthrough in the search for creating scalable sources of photons using single QDs and much excitement is envisioned. However, the lack of appropriate theoretical descriptions becomes very challenging and the development of new medium-dependent models are required to better describe the light-matter interactions and photon wave functions.

In quantum material systems such as solids, the interaction with the environment is inevitable. The biexcitons, excitons, and cavity modes interact with their phonon and thermal reservoirs,[24,25,29,30] which can have a substantial influence on the wave function of the emitted photon pairs. In the biexciton decay, the entanglement depends on the "indistinguishability" between x-polarized and y-polarized photon pairs, namely, the overlap of their wave functions. Therefore, the precise form of the wave functions of the emitted photon pairs is ultimately required. Here we present rigorous and physically intuitive analytical expressions for the wave function of the emitted photon pairs in the biexciton-exciton cascade decay using the Weisskopf-Wigner approximation for coupling to the environment. Extending previous approaches,[22] we consider finite exciton and biexciton level broadenings and the damping of the leaky cavity mode. We further apply a method for optimizing the entanglement using a simple spectral filter[13] and find impressive entanglement values even with realistic parameters and a sizable anisotropic energy exchange.

II. THEORY

We consider a QD embedded in a photonic-crystal cavity having two orthogonal polarization modes of frequency ω_c^x

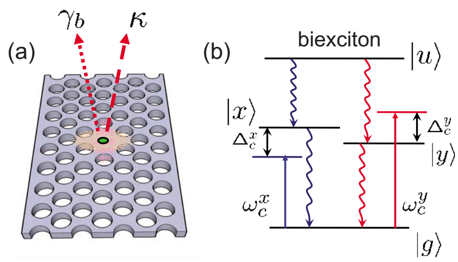

FIG. 1. (Color online) (a) Schematic of the planar photonic crystal containing a single QD and (b) the resulting energy-level diagram for cavity-QED-assisted generation of entangled photons in the biexciton-exciton cascade decay. The biexciton state $|u\rangle$ decays to the ground state $|g\rangle$ via intermediate exciton state $|x\rangle$ or $|y\rangle$, emitting an x-polarized or y-polarized photon pair. The x-polarized and y-polarized cavity modes are coupled with the $|x\rangle \to |g\rangle$ and $|y\rangle \to |g\rangle$ transition, respectively. The vertical decays represent the leaky cavity mode decay (κ) and the spontaneous decay from the background radiation modes (γ_b) (above the photonic-crystal slab light line).

and ω_c^y, which can be realized and tuned experimentally using electron-beam lithography and, for example, atomic force microscopy (AFM) oxidization techniques.[31] The exciton states, $|x\rangle$ and $|y\rangle$, have an anisotropic-exchange energy difference δ_x. The cavity modes are coupled with the exciton to ground-state transition but spectrally decoupled from the biexciton state because of the relatively large biexciton binding energy, $\Delta_{xx} \gg$ cavity coupling. The schematic arrangement of the system is shown in Fig. 1. For simplicity, we consider the emission of x-polarized photon pair but the formalism and results also apply to the y-polarized photons as well. The Hamiltonian for the emission of x-polarized photon pair, in the interaction picture, can be written as

$$H_I(t) = \hbar \Bigg[g|x\rangle\langle g|\hat{a}_c e^{i\Delta_c^x t} + \sum_{k\neq c} \Omega_{uk}|u\rangle\langle x|\hat{a}_k e^{i(\omega_{ux}-\omega_k)t} + \sum_{l\neq c} \Omega_{gl}|x\rangle\langle g|\hat{a}_l e^{i(\omega_x-\omega_l)t} + \sum_{m\neq c} \Omega_{cm}\hat{a}_c^\dagger \hat{a}_m e^{i(\omega_c^x-\omega_m)t}\Bigg], \tag{1}$$

plus a Hermitian conjugate term, where $\omega_{ux}=\omega_u-\omega_x$, $\Delta_c^x = \omega_x-\omega_c^x$, and $\hat{a}_i$ is the field operators with $\hat{a}_c$ the cavity mode operator. Here, Ω_{uk}, Ω_{gl}, and Ω_{cm} represent the couplings to the environment from the biexciton, exciton, and cavity mode; g is the coupling between the exciton and cavity mode; and ω_k, ω_l, ω_m, ω_u, and ω_x are the frequency of the photon emitted from the biexciton and exciton, the frequency of the photon leaked from cavity, and the frequency of the biexciton and exciton, respectively. We consider a system that is optically pumped in such a way as to have an initially excited biexciton with no cavity photons, thus the state of the system at any time t can be written as

$$|\psi(t)\rangle = c_1(t)|u,0\rangle + \sum_k c_{2k}(t)|x,0\rangle|1_k\rangle + \sum_k c_{3k}(t)|g,1\rangle|1_k\rangle + \sum_{k,l} c_{4kl}(t)|g,0\rangle|1_k,1_l\rangle + \sum_{k,m} c_{5km}(t)|g,0\rangle|1_k\rangle|1_m\rangle. \tag{2}$$

The different terms in the state vector $|\psi\rangle$ represent, respectively, the dot is in the biexciton state with zero photons in the cavity, the dot is in the exciton state after radiating one photon, the dot is in ground state with one photon in cavity mode, the dot is in the ground state after radiating two photons, and the dot is in ground state after one photon is radiated from the biexciton and the other is emitted via the leaky cavity mode.

By using the Schrödinger equation, the equations of motion for the probability amplitudes are

$$\dot{c}_1(t) = -i\sum_k \Omega_{uk} c_{2k}(t) e^{i(\omega_{ux}-\omega_k)t}, \tag{3}$$

$$\dot{c}_{2k}(t) = -i\Omega_{uk}^* c_1 e^{-i(\omega_{ux}-\omega_k)t} - igc_{3k}(t)e^{i\Delta_c^x t} - i\sum_l \Omega_{gl} c_{4kl}(t) e^{i(\omega_x-\omega_l)t}, \tag{4}$$

$$\dot{c}_{3k}(t) = -igc_{2k}(t)e^{-i\Delta_c^x t} - i\sum_m \Omega_{cm} c_{5km}(t) e^{i(\omega_c^x-\omega_m)t}, \tag{5}$$

$$\dot{c}_{4kl}(t) = -i\Omega_{gl}^* c_{2k}(t) e^{-i(\omega_x-\omega_l)t}, \tag{6}$$

$$\dot{c}_{5km}(t) = -i\Omega_{cm}^* c_{3k}(t) e^{-i(\omega_c^x-\omega_m)t}. \tag{7}$$

Applying the Weisskopf-Wigner approximation,[32] then Eqs. (3)–(5) simplify to

$$\dot{c}_1(t) = -\gamma_1 c_1(t), \tag{8}$$

$$\dot{c}_{2k}(t) = -i\Omega_{uk}^* c_1(t) e^{-i(\omega_{ux}-\omega_k)t} - igc_{3k}(t)e^{i\Delta_c^x t} - \gamma_2 c_{2k}(t), \tag{9}$$

$$\dot{c}_{3k}(t) = -igc_{2k}(t)e^{-i\Delta_c^x t} - \kappa c_{3k}(t), \tag{10}$$

where $\kappa = \pi|\Omega_{cm}|^2$ is the half width of the cavity mode, and γ_1 and γ_2 are the half widths of the biexciton and exciton levels, respectively. We note that γ_1 and γ_2 can include both radiative and nonradiative broadening and for QDs, $\gamma_1 \approx 2\gamma_2$. Moreover, the radiative half width of biexciton will be sum of its spontaneous decay rates in the exciton states $|x\rangle$ and $|y\rangle$; if the decay rate of the biexciton in $|x\rangle$ and $|y\rangle$ are equal, the radiative half width of biexciton will be $2\pi|\Omega_{uk}|^2$. The radiative half width of the exciton $|x\rangle$ is given by $\gamma_b = \pi|\Omega_{gl}|^2$. We next solve Eqs. (6)–(10) to obtain c_{4kl} and c_{5km} using the Laplace transform method. The probability amplitudes for two-photon emission, in the long-time limit, are given by

$$c_{4kl}(\infty) = \frac{\Omega_{uk}^*}{(\omega_k+\omega_l-\omega_u+i\gamma_1)} \times \frac{\Omega_{gl}^*(\omega_l-\omega_c^x+i\kappa)}{(\omega_l-\omega_x+ig_+)(\omega_l-\omega_x+ig_-)}, \tag{11}$$

$$c_{5km}(\infty) = \frac{\Omega_{uk}^*}{(\omega_k + \omega_m - \omega_u + i\gamma_1)} \times \frac{g\Omega_{cm}^*}{(\omega_m - \omega_x + ig_+)(\omega_m - \omega_x + ig_-)}, \quad (12)$$

where $g_\pm = 0.5[\kappa + \gamma_2 - i\Delta_c^x \pm i\sqrt{4g^2 - (\kappa - \gamma_2 - i\Delta_c^x)^2}]$. In the case of no cavity coupling, namely, $g=0$, the photons will be emitted spontaneously from the dot and we obtain a limiting form

$$c_{4kl}(\infty) = \frac{\Omega_{uk}^* \Omega_{gl}^*}{(\omega_k + \omega_l - \omega_u + i\gamma_1)(\omega_l - \omega_x + i\gamma_2)}, \quad (13)$$

which is the two-photon emission probability amplitude from a cascade in free space, in agreement with results of Akopian *et al.*[13] Thus, the influence of the cavity is determined by $g_\pm$, as one might expect. Next, the optical spectrum of the *spontaneously* emitted photon pair, via *radiation* modes (above the photonic-crystal light line), is given by $S_r(\omega_k, \omega_l) = |c_{4kl}(\infty)|^2$, where

$$S_r(\omega_k, \omega_l) = \frac{|\Omega_{uk}|^2}{[(\omega_k + \omega_l - \omega_u)^2 + \gamma_1^2]} \times \frac{|\Omega_{gl}|^2 |(\omega_l - \omega_c^x + i\kappa)|^2}{|(\omega_l - \omega_x + ig_+)(\omega_l - \omega_x + ig_-)|^2}. \quad (14)$$

Similarly, the spectrum of the photon pair with one photon emitted spontaneously from the biexciton and the other photon emitted via the leaky cavity mode [cf. Fig. 1], is $S_c(\omega_k, \omega_m) = |c_{5km}(\infty)|^2$, where

$$S_c(\omega_k, \omega_m) = \frac{|\Omega_{uk}|^2}{[(\omega_k + \omega_m - \omega_u)^2 + \gamma_1^2]} \times \frac{g^2 |\Omega_{cm}|^2}{|(\omega_m - \omega_x + ig_+)(\omega_m - \omega_x + ig_-)|^2}. \quad (15)$$

The spectral functions $S_r(\omega_k, \omega_l)$ and $S_c(\omega_k, \omega_m)$ represent the joint probability distribution and thus the integration over the one frequency variable gives the spectrum at the other frequency. For example, the spectrum of the photon coming from the spontaneous decay of the exciton decay will be $S_r(\omega_l) = \int_{-\infty}^{\infty} S_r(\omega_k, \omega_l) d\omega_k$ and the spectrum of photon emitted via cavity mode is $S_c(\omega_m) = \int_{-\infty}^{\infty} S_c(\omega_k, \omega_m) d\omega_k$. One obtains

$$S_r(\omega_l) = \frac{|\Omega_{gl}|^2 |(\omega_l - \omega_c^x + i\kappa)|^2}{|(\omega_l - \omega_x + ig_+)(\omega_l - \omega_x + ig_-)|^2}, \quad (16)$$

$$S_c(\omega_m) = \frac{g^2 |\Omega_{cm}|^2}{|(\omega_m - \omega_x + ig_+)(\omega_m - \omega_x + ig_-)|^2}, \quad (17)$$

which is similar to the radiation mode and cavity mode emitted spectra reported by Cui and Raymer[33] and by Hughes and Yao.[34] From Eqs. (16) and (17), the photon emitted from the exciton decay (second emitted photon) has a two-peak spectrum; these spectral peaks appear at the frequencies $\frac{1}{2}(\omega_x + \omega_c^x \pm \delta\omega)$, where $\delta\omega \approx \sqrt{4g^2 + \Delta_c^{x2} - (\kappa - \gamma_2)^2}$ is the splitting between the peaks. In a dressed-state picture, these spectral peaks correspond to the two polariton states in the strong cavity regime, $g \gg (\kappa, \gamma_2)$.[22]

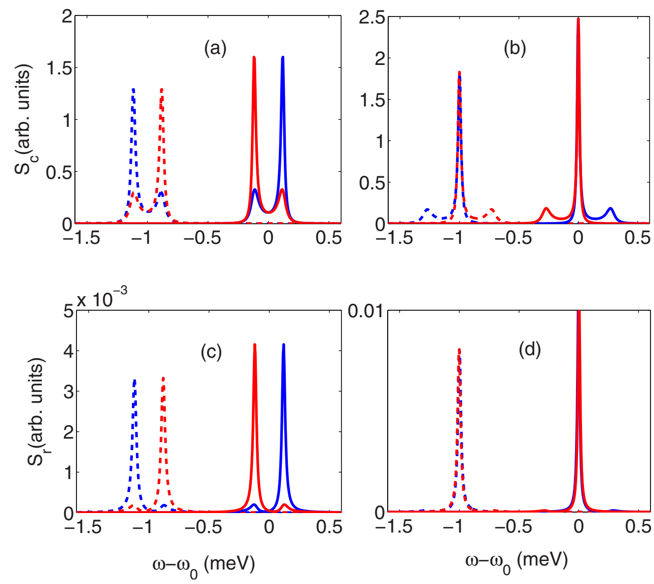

FIG. 2. (Color online) The spectra, $S_c(\omega)$ and $S_r(\omega)$, of the generated photons in the biexciton-exciton cascade decay for $\delta_x = 0.1$ meV, $\Delta_{xx} = 1.0$ meV, $\gamma_1 = 2\gamma_2 = 0.004$ meV, $\gamma_b = \pi|\Omega_{uk}|^2 = \pi|\Omega_{gl}|^2 = 0.05$ μeV (Ref. 28), $\kappa = 0.05$ meV, $g = 0.11$ meV, and $\omega_0 = (\omega_x + \omega_y)/2$. In (a) and (b), one photon is emitted from the biexciton decay and the other is emitted via the leaky cavity mode; in (c) and (d), both photons are radiated from the biexciton and exciton states via spontaneous (radiation-mode) decay. The other parameters are as follows: for (a) and (c), $\Delta_c^x = -\Delta_c^y = \delta_x$ and for (b) and (d), $\Delta_c^x = -\Delta_c^y = -0.175$ meV. The x-polarized photons are shown in blue and the y polarized are shown in red; also, the solid (right) curves are for photons generated in the exciton decay and the dotted curves are for photons generated in the biexciton decay.

From the above discussion, the state of the "photon pair" emitted from both the $|x\rangle$-exciton and $|y\rangle$-exciton branches is given by

$$|\psi(\infty)\rangle = \sum_{k,l} c_{4kl}(\infty)|1_k, 1_l\rangle_x |0\rangle_x + \sum_{k,m} c_{5km}(\infty)|1_k\rangle_x |1_m\rangle_x + \sum_{k,l} d_{4kl}(\infty)|1_k, 1_l\rangle_y |0\rangle_y + \sum_{k,m} d_{5km}(\infty)|1_k\rangle_y |1_m\rangle_y, \quad (18)$$

where in each term the first ket represents the combined state of the biexciton and the exciton reservoirs, the second ket represents the state of the cavity reservoir, and the ket suffix labels the polarization. The coefficients $c_{ijk}(\infty)$ are given by Eqs. (11) and (12). For the same cavity coupling g, the coefficients, d_{ijk}, are given by the Eqs. (11) and (12) after replacing ω_x, ω_c^x, and Δ_c^x with ω_y, ω_c^y, and $\Delta_c^y = \omega_y - \omega_c^y$, respectively.

III. RESULTS AND OPTIMIZING THE ENTANGLEMENT

There are two possible decay channels for generating a photon pair. In Figs. 2(a) and 2(b), we show two examples of the spectra for the photon pair when one photon is emitted

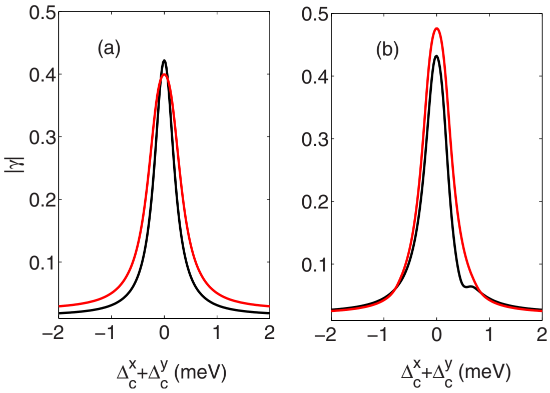

FIG. 3. (Color online) The amplitude of the off-diagonal element of the density matrix for filtered photon pairs where δ_x is fixed at 0.1 meV. In (a) and (b) is shown the unfiltered and filter cases, respectively. The black curve represents $\Delta_c^x-\Delta_c^y=2\delta_x$ (case 1) and the red curve represents $\Delta_c^x-\Delta_c^y=-0.35$ meV (case 2); the other parameters are the same as in Fig. 2. The filter function corresponds to two spectral windows of width $w=0.2$ meV centered at ω_x^- and $\omega_u-\omega_x^-$. Note that $\Delta_c^x+\Delta_c^y=0$ corresponds to the optimal conditions for generating entangled photon pairs as shown in Fig. 2.

from biexciton decay and the second is emitted (leaked) via the cavity mode. The spectra of photon pairs emitted in the biexciton and exciton radiative decay are shown in Figs. 2(c) and 2(d). Depending on the detunings between the frequency of the cavity field and the frequency of the excitons, $\Delta_c^{x,y}$, the x-polarized photon pair and y-polarized photon pair can be degenerate in energy. The spectra of the emitted x- and y-polarized photons, in the strong-coupling regime, show peaks at the frequencies $\omega_k \approx \omega_u-\omega_{x,y}^{\pm}$ and $\omega_{l/m}\approx\omega_{x,y}^{\pm}$, where $\omega_{x,y}^{\pm}=\frac{1}{2}[\omega_c^{x,y}+\omega_{x,y}\pm\sqrt{(\Delta_c^{x,y})^2+4g^2}]$ are the frequencies of the polariton states. The polarization-entangled photon pairs can be generated by making the emitted x-polarized and y-polarized photon pairs degenerate. For the positive (negative) values of $\Delta_c^{x,y}$, the peaks in the spectrum corresponding to $\omega_k\approx\omega_u-\omega_{x,y}^+$ and $\omega_{l/m}\approx\omega_{x,y}^+$ ($\omega_k\approx\omega_u-\omega_{x,y}^-$ and $\omega_{l/m}\approx\omega_{x,y}^-$) are stronger and the probability of generating photons for these frequencies is increased. Therefore, a large probability of generating degenerate photon pairs can be achieved by overlapping these stronger peaks in the spectrum. There are three possible coupling cases of interest that can do this.[22] *Case 1:* by making both x-polariton states and y-polariton states degenerate, $\omega_x^{\pm}=\omega_y^{\pm}$ which can be achieved with $\Delta_c^x=-\Delta_c^y=\delta_x$ [see Figs. 2(a) and 2(c)]; *case 2:* by making one of the x-polariton states degenerate to the other y-polariton state [$\omega_x^-=\omega_y^+$, see Figs. 2(b) and 2(d); or $\omega_x^+=\omega_y^-$] when Δ_c^x and Δ_c^y are of opposite sign; *case 3:* by making $\omega_x^+=\omega_y^+(\omega_x^-=\omega_y^-)$ when both Δ_c^x and Δ_c^y are positive (negative). Optimum entanglement is achieved from case 1 and case 2 above for $\Delta_c^x=-\Delta_c^y$, which we example in Figs. 2 and 3.

We stress that our calculated spectra are significantly different to those predicted previously using a dressed-state picture where the latter uses simple Lorentzian linewidths for each state.[22] Moreover, in the strong-coupling regime, the cavity-assisted-generated photon pairs (S_c) completely dominates the spontaneously emitted photons (S_r) and by several orders of magnitude. This effect is similar to the cavity-feeding process that occurs for an off-resonant cavity mode[34] where the leaky cavity mode emission dominates the spectrum. Thus, one can basically ignore the contribution from S_r.

The entanglement can be distilled by using frequency filters with a small spectral window w centered at the frequencies of degenerate peaks in the spectrum of x-polarized and y-polarized photons. Subsequently, the response of spectral filter can be written as a projection operator of the following form:

$$W(\omega_k,\omega_m)=\begin{cases}1, & \text{for } |\omega_k-\omega_u+\omega_{x,y}^{\pm}|<w,\\ 1, & \text{for } |\omega_m-\omega_{x,y}^{\pm}|<w,\\ 0, & \text{otherwise.}\end{cases} \tag{19}$$

After operating on the wave function of the emitted photons [Eq. (18)], by the spectral function $W(\omega_k,\omega_m)$ and tracing over the energy states,[13] we get the reduced density matrix of the filtered photon pairs in the polarization basis. We consider the photon pairs in which one photon is emitted from the biexciton decay and the other is emitted by the leaky cavity mode; in fact we can easily neglect the spontaneous emission of both biexciton and exciton photons as discussed above. The normalized off-diagonal element of the density matrix of photons is given by[13]

$$\gamma=\frac{\int\int c_{5km}^*(\infty)d_{5km}(\infty)W d\omega_k d\omega_m}{\int\int |c_{5km}(\infty)|^2 W d\omega_k d\omega_m+\int\int |d_{5km}(\infty)|^2 W d\omega_k d\omega_m}. \tag{20}$$

The *concurrence*, which is a quantitative measure of entanglement for the state of the filtered photon pair is given by $C=2|\gamma|$.[24] The photons are thus maximally entangled when $|\gamma|=0.5$. In Fig. 3, the value of $|\gamma|$ is plotted for two different cases of degenerate x-polarized and y-polarized photon pairs, corresponding to Figs. 2(a) and 2(b); δ_x and $\Delta_c^x-\Delta_c^y$ are fixed, while $\Delta_c^x+\Delta_c^y$ is changed, e.g., by temperature or gas tuning;[31,35] both (a) unfiltered and (b) filtered values are shown. The spectral filter has negligible effect on case 1 but it improves the concurrence of case 2 significantly. After filtering, the generated photons, when both polariton states of the x-polarized and y-polarized photons are degenerate [see Fig. 2(a)], have a smaller entanglement than the generated photons when one x-polarized polariton state, ω_x^-, is degenerate with one y-polarized polariton state, ω_y^+, [see Fig. 2(b)]. However, the photon source operating under the conditions of Fig. 2(a) is a *deterministic entangled photon source*, while the photon source operating under the conditions of Fig. 2(b)—and using a spectral filter—is a probabilistic photon sources as there is some probability of generating nondegenerate photon pairs. In both cases, we get pronounced concurrence values of greater than 0.9.

Finally, we discuss the criteria for achieving efficient entanglement using the photonic-crystal cavity scheme. In general, one desires to be in the strong-coupling regime to overcome the exchange splitting, thus the required conditions are

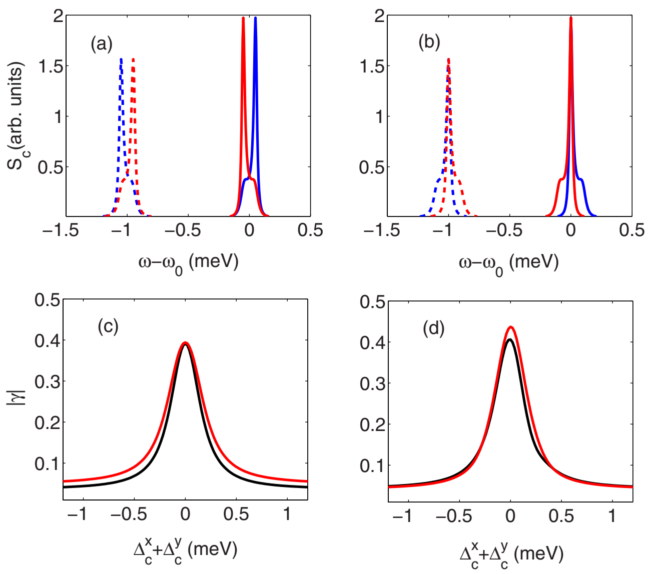

FIG. 4. (Color online) [(a) and (b)] As in Figs. 2(a) and 2(b) but with $g=\kappa=\delta_x=0.05$ meV. [(c) and (d)] As in Figs. 3(a) and 3(b) but with $g=\kappa=\delta_x=0.05$. The black curve represents $\Delta_c^x-\Delta_c^y=2\delta_x$ (case 1) and the red curve represents $\Delta_c^x-\Delta_c^y=-0.1$ meV (case 2). The filter function corresponds to two spectral windows of width $w=0.1$ meV, centered at ω_x^- and $\omega_u-\omega_x^-$.

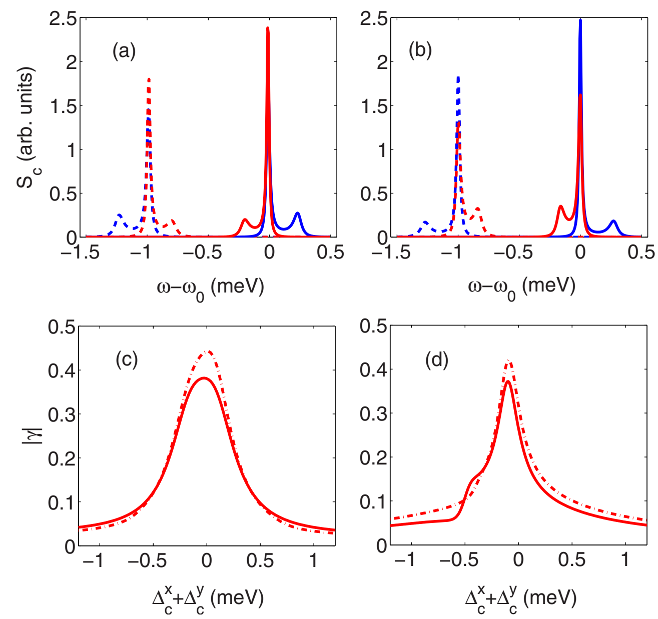

FIG. 5. (Color online) [(a) and (b)] Same as in Fig. 2(b) but with $g_x=0.11$ meV and $g_y=0.08$ meV for (a) $\Delta_c^x=-\Delta_c^y=-0.122$ meV, and for (b) $\Delta_c^x=-0.175$ meV and $\Delta_c^y=0.07$ meV. [(c) and (d)] The values of $|\gamma|$ for generated photons by using two possible tuning methods, in (c) by changing $\Delta_c^x+\Delta_c^y$ for $\Delta_c^x-\Delta_c^y=0.245$ meV and in (d) by changing Δ_c^y for $\Delta_c^x=-0.175$ meV. The chain curves represent results for filtered photons and the solid curves represent results for unfiltered photons; the filter function corresponds to two spectral windows of width $w=0.2$ meV, centered at ω_x^- and $\omega_u-\omega_x^-$.

$g>\kappa$ and $g>\delta_x/2$. To gain insight into a smaller g situation, we show in Fig. 4, the spectra and entanglement that occurs for $g=\kappa$ and for smaller values of δ_x. For the spectra (a) and (b), it is clear that the indistinguishability of the x-polarized and y-polarized pairs is increased, yet in (c) and (d) we see that impressive entanglement values can still be achieved, even without a filter. In addition, use of a spectral filter cannot improve the entanglement significantly in these conditions. Thus we believe that the general cavity improvement could be significant in the context of generated entangled photon pairs and that these values are achievable using realistic and experimentally accessible parameters.

In the QD-photonic-crystal cavity system, there is some possibility that the coupling strengths of exciton with the x-polarized mode, g_x, and with the y-polarized mode, g_y, could be different so that $g=g_x\neq g_y$. The difference between the coupling strengths may occur because of the anisotropy in the QD system or a misalignment between the dot and the positions of the cavity field antinodes. In such experimental situations, it is not possible to satisfy the conditions of case 1, which had previously made both x-polariton states and y-polariton states degenerate. However, the conditions of case 2, which make one of the x-polariton state degenerate to the other y-polariton state can still be achieved in two different ways: either for $\Delta_c^x=-\Delta_c^y$ [see Fig. 5(a)] or for $\Delta_c^x\neq-\Delta_c^y$ [see Fig. 5(b)]. The values of $|\gamma|$ are shown in Figs. 5(c) and 5(d) where the values of entanglement are only slightly less than the values achieved in Fig. 3. For tuning, in Fig. 5(c), the value of $\Delta_c^x+\Delta_c^y$ is changed while keeping $\Delta_c^x-\Delta_c^y$ constant, which can be achieved by temperature or gas tuning methods.[35] In Fig. 5(d), we change Δ_c^y while keeping Δ_c^x fixed, which is possible—as mentioned earlier—by changing the frequencies of cavity modes independently using AFM oxidation methods.[31]

IV. CONCLUSION

In conclusion, we have derived and exploited general analytical results for the wave functions of the emitted photon pairs from a QD embedded in a photonic-crystal cavity that supports quasidegenerate cavity modes. In particular, we have included finite exciton- and biexciton-level broadenings, and the damping of the leaky cavity modes, and shown that these relaxation mechanisms should be included to connect to realistic experiments. Finally, we have also discussed a method for optimizing and measuring the entanglement between the emitted photons using a simple spectral filter.

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